

# Development of a Digital Twin Model for Cart-Pendulum Systems

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**Abstract.** Digital twin models can combine mathematical models and describe dynamics of machines visually in details. Control mechanism can be designed and planned based on such models by considering the overall machine performance and machines operation. Therefore, machine design can be optimized accordingly in terms of the standards on effectiveness, efficiency and safety. In a long run, real-time condition monitoring on machines can be achieved by employing a digital twin which ensures the accuracy of the diagnosis as well as reduces time consuming process and the need of professional knowledge. The research in this paper presents a development method of digital twin model for cart-pendulum systems which is based on MATLAB/Simulink Multi-body simulation. The effectiveness of the modeling method is verified in test platform.

**Keywords;** digital twin, dynamics modeling, control systems, simulation, cart-pendulum, industrial applications

## 1. Introduction

With the rapid development and implementation of information technology and automation, smart machines grow significantly and they would power the new industrial revolution that change will how machine builders design and how manufacturers operate. To remain competitive and profitable, manufacturers expect that plants and machines are better connected, more efficient, more flexible, and safe in the future decades. These are the key features of smart machines [1]. There are two trends throughout global industry which are Industry 4.0 and Industrial Internet of Things (IoT). The former is on how to develop the manufacturing process, and the latter is on how the devices are

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connected and big data analysis. The basis for both of these trends is to enhance networked resources so that distributed intelligence can lead to improved visibility and management of production [2].

Therefore, in the context of developing and manufacturing complex smart machines, digitalization begins with creating a digital model. This digital model, or digital twin, should describe, define, capture, and analyze how the product is expected to perform. The digital twin is often described as a digital replica of different assets, processes, and systems in a business that can be used in a number of ways. A comprehensive digital twin consists of many mathematical models and virtual representations that encompass the asset's entire life-cycle from ideation, through realization and utilization and all its constituent technologies [3]. A machine digital twin will typically include electronics and software simulations; finite element structural, flow, and heat transfer models; and motion simulations. This comprehensive computerized model of the machine enables almost 100 percent of virtual validation and testing of the product under design, which minimizes the need for physical prototypes, reduces the amount of time needed for verification, improves quality of the final manufactured product, and enables faster reiteration in response to customer feedback [4][5].

Cart-Pendulum systems have numerous industrial applications in robotics, aerospace, transportation, manufacturing, and etc. Such systems are nonlinear in nature, and however, they can be simplified into linear system around the equilibrium under certain circumstances. Given the significance of cart-pendulum systems, it is desirable to develop a digital twin model which could assist practical engineers in system design and performance evaluation. In addition, the digital twin model can also work a reference model in condition monitoring, which provides the data basis for normal operation. The fault detection, diagnosis, and accommodation can be achieved by such models, and the source and scale of faults can be presented visually [6].

The rest of this research work is structured as follows. In Section 2, the equations of motion of cart-pendulum systems are presented. In Section 3, a digital twin model is developed in MATLAB Simcape. In Section 4, the design of positioning control of cart is achieved with vibration suppression. In Section 5, the real-time data from test platform is imported into digital twin model for performance evaluation, and Section 6 concludes this paper.

## 2. Dynamics of Cart-Pendulum Systems

A simple pendulum on a cart traveling on a friction-less track is shown in Fig. 1.

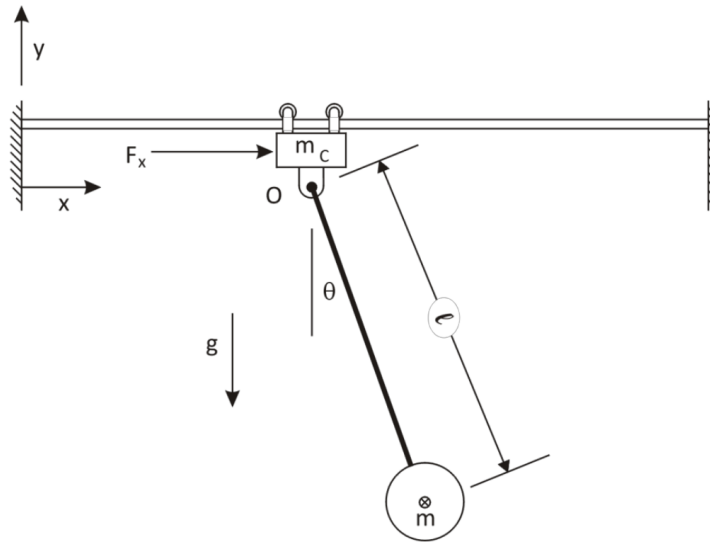


Fig 1. A Simple Cart-Pendulum System

The cart is driven back and forth on the track by an external force acting along the path of the track. The pendulum is starting to look like an overhead or gantry crane. By Lagrangian formulation, the equations to describe the dynamics are as follows:

$$(m_c + m) \cdot \ddot{x}_c + m \cdot l \cdot (\ddot{\theta} \cdot \cos\theta - \dot{\theta}^2 \cdot \sin\theta) = F_x \quad (1)$$

$$\ddot{x}_c \cdot l \cdot \cos\theta + \ddot{\theta} \cdot l^2 + g \cdot l \cdot \sin\theta = 0 \quad (2)$$

where  $x_c$  is the position of cart along with the rail;  $m_c$  and  $m$  are the mass of cart and pendulum, respectively;  $l$  is the length of cord;  $\theta$  is the angle between the cord and the direction of gravity;  $g$  is the gravity;  $F_x$  is the external force applied on cart.

## 3. Digital Twin Model of Cart-Pendulum

A digital twin model for cart-pendulum systems is developed in MATLAB/Simulink, using Simcape Multibody. The motion of cart is simulated by a prismatic joint and the revolution of pendulum by revolute joint. The details of the model structure are shown in Fig. 2. A common control task in cart-pendulum systems is cart positioning with vibration suppression on pendulum. To compare the performance of different control

designs, the model in Fig. 2 have two independent control inputs. The upper block is showing the cart-pendulum system without control, which use step input with a low-pass filter; the lower block is the cart-pendulum system that has feed-forward control on cart positioning.

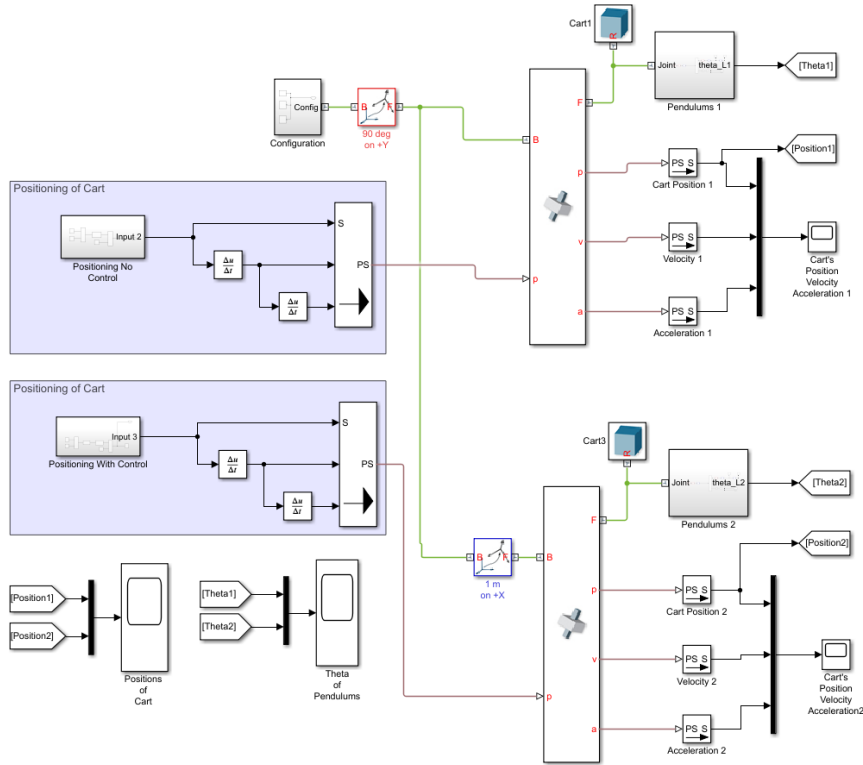


Fig 2. MATLAB Simcape Blocks for Cart-Pendulum Systems

#### 4. Control Design Using Digital Twin Model

The technique for the design of time-delay filters for single-mode systems can be extended to multiple-mode systems by cascading the time-delay filters designed for each mode. This will not lead to the smallest number of delay times to cancel the dynamics of the two underdamped modes. Fig. 3 illustrates the cascaded time-delay filter for a multi-mode system.

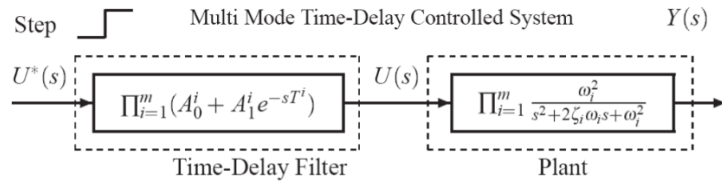


Fig 3. Input Shaping Control for Vibration Suppression

$$A_0^i = \frac{\exp\left(\frac{\zeta_i \pi}{\sqrt{1-\zeta_i^2}}\right)}{\exp\left(\frac{\zeta_i \pi}{\sqrt{1-\zeta_i^2}}\right) + 1} \tag{3}$$

$$A_1^i = \frac{1}{\exp\left(\frac{\zeta_i \pi}{\sqrt{1-\zeta_i^2}}\right) + 1} \tag{4}$$

$$T^i = \frac{\pi}{\omega_i \sqrt{1-\zeta_i^2}} \tag{5}$$

The control results are shown in Fig. 4 and Fig. 5. It can be seen that the input-shaping method can achieve similar control performance with step input with low pass filter. However, the input shaping method has significant improvement in vibration suppression.

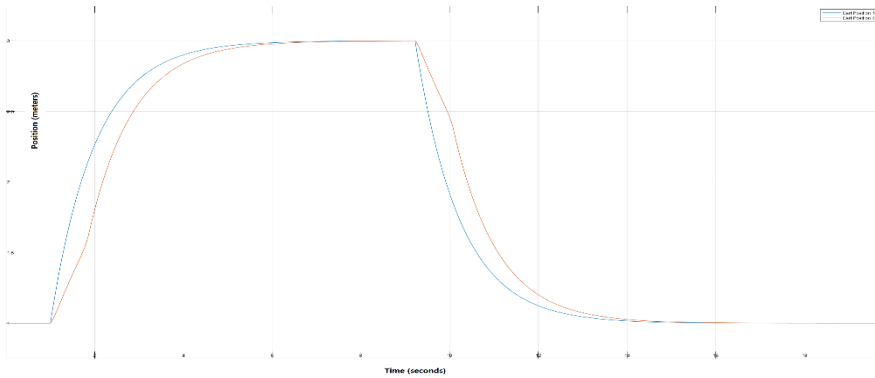


Fig 4. Cart-Positions in Simulation Result

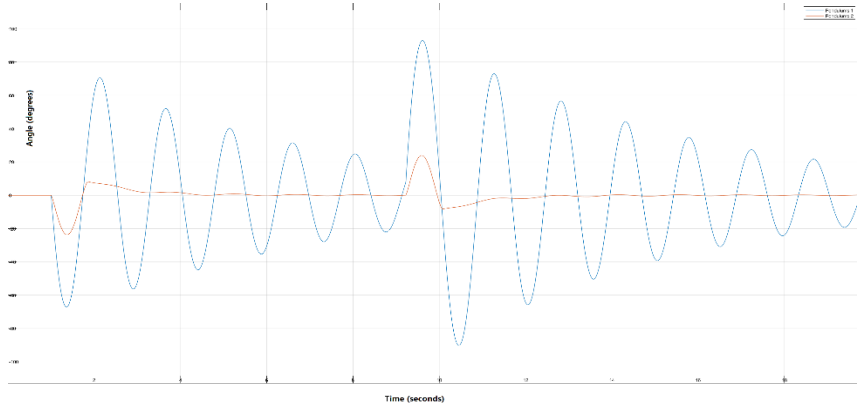


Fig 5. Pendulum Angles in Simulation Result

### 5. Condition Monitoring with Digital Twin Model

The test platform adopted in this research is based on B&R X20 systems that support Modbus communication between PLC and PC. As shown in Fig. 6, the communication between digital twin model and test platform can be built to collect the measure data for analysis.

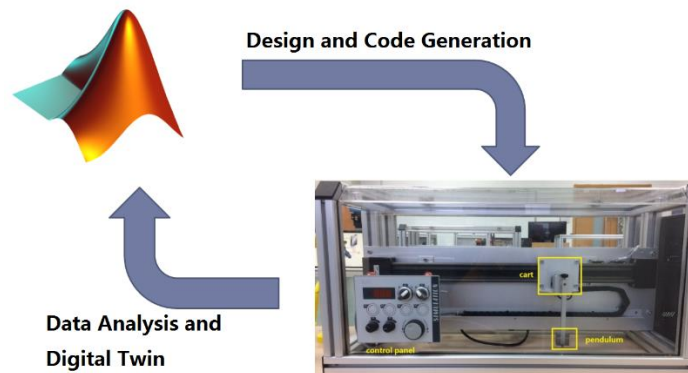


Fig 6. Interaction Between Digital Twin Model and Real System

The collected data can be fed into the digital twin model as shown in Fig. 7.

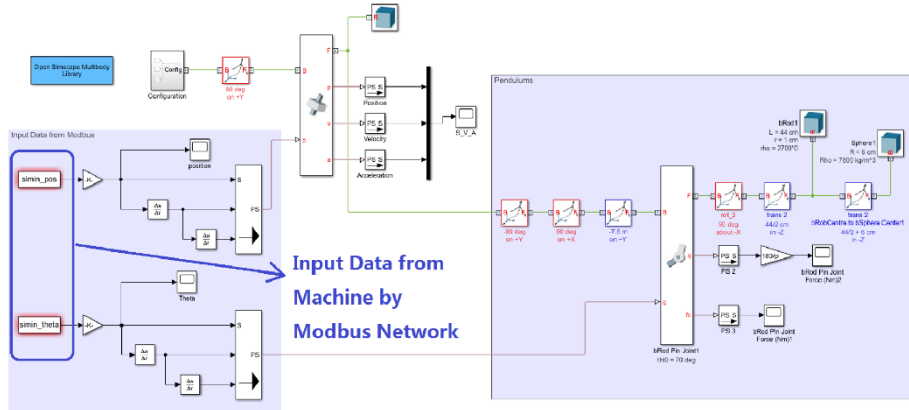


Fig 7. Digital Twin Model Interface with Real System

The animation of controlled process can be generated by the digital twin as shown in Fig. 8.

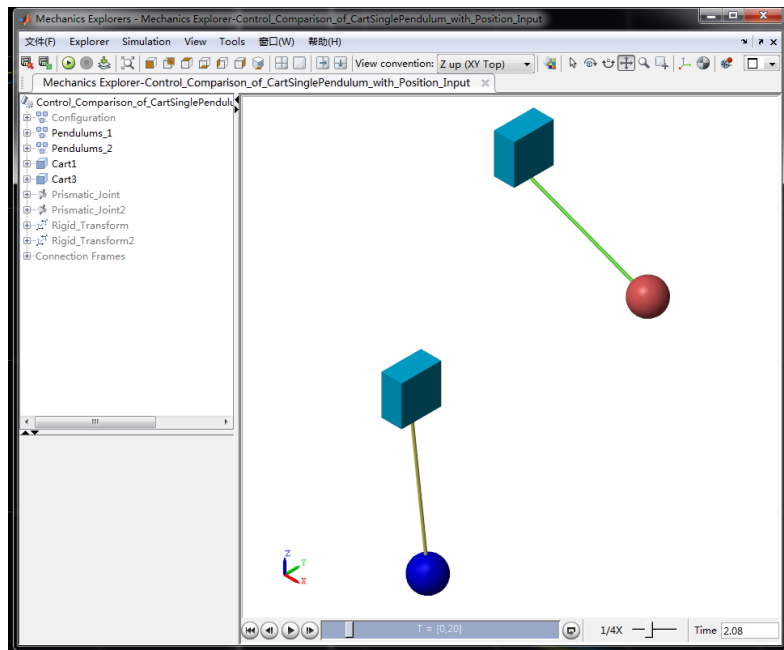


Fig 8. Animation Generated by Digital Twin Model with Measured Data

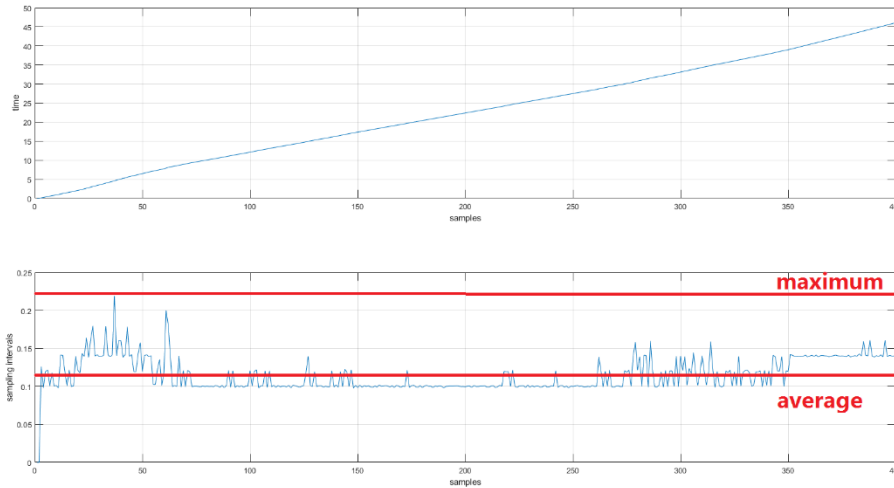


Fig 9. The Effect of Time Delay in Communication

However, the simulation results show that the time delay effect has influence on smoothness of visual data. With the increasing of machine numbers, the visual presentation on the machine operation might be degraded. A more robust mechanism in communication is desirable, which could be the future research work.

## 6. Conclusions

- The research proposed a digital twin modeling method for cart-pendulum systems, and the method can be used for dynamics analysis, control design, real-time monitoring, and even fault diagnosis and accommodation in future works.
- The control design for vibration suppression in cart-pendulum systems has been achieved, and the result shows satisfactory performance in positioning task.
- The effectiveness of digital-twin model method for online monitoring has been verified by the platform that is built by industrial systems. The model can make the fault diagnosis process more smooth with details by visual presentation.
- The issue during implementation which is brought by network communication has been addressed.



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